PHOTOGRAPHIC DEFINITION.

I.

IT is a matter of some interest to determine what are the limits to the definition obtainable in photographs. In examining this question, three distinct classes of problems present themselves—namely:—

(1) Those depending on the wave-length of light, and the action of a perfect lens on such wave-lengths.

(2) The various aberrations of real lenses.

(3) The qualities of the different sensitive surfaces on which the pictures are formed.

Taking these divisions of the subject in the order given, I will inquire first what is the limit to photographic definition on the supposition that the lens has no aberration of any kind, *i.e.* that all the waves which reach it from any point arrive at the image of that point in the same phase.

The image thus formed consists, as is well known, of a bright disk surrounded by alternate dark and bright rings, the intensity of the illumination of the rings decreasing rapidly at each successive ring, reckoning outwards from the centre.

In order that the images of two neighbouring points may appear separated from one another, the central disks of their images ought not to overlap. If the disks are just in contact, it is possible that they would appear as a double object in the photograph, and this may be taken as the limit of the defining power of a lens. (See Airy "On Light," and Lord Rayleigh "On the Theory and Manufacture of Diffraction Gratings," *Phil. Mag.*, 1874.)

But, in ordinary photography, objects at very various distances have to be simultaneously represented, and it is to the definition attainable under these circumstances that I wish now to direct attention.

On referring to the papers above-mentioned, it will be seen that the diameter of the central disk is

$$1{\cdot}219\,\frac{\lambda F}{A},$$

where λ is the wave-length of light,

F the focal length of the lens,

A the aperture of the lens.

This gives the effective diameter of the image of a point truly in focus when not far removed from the axis of the lens.

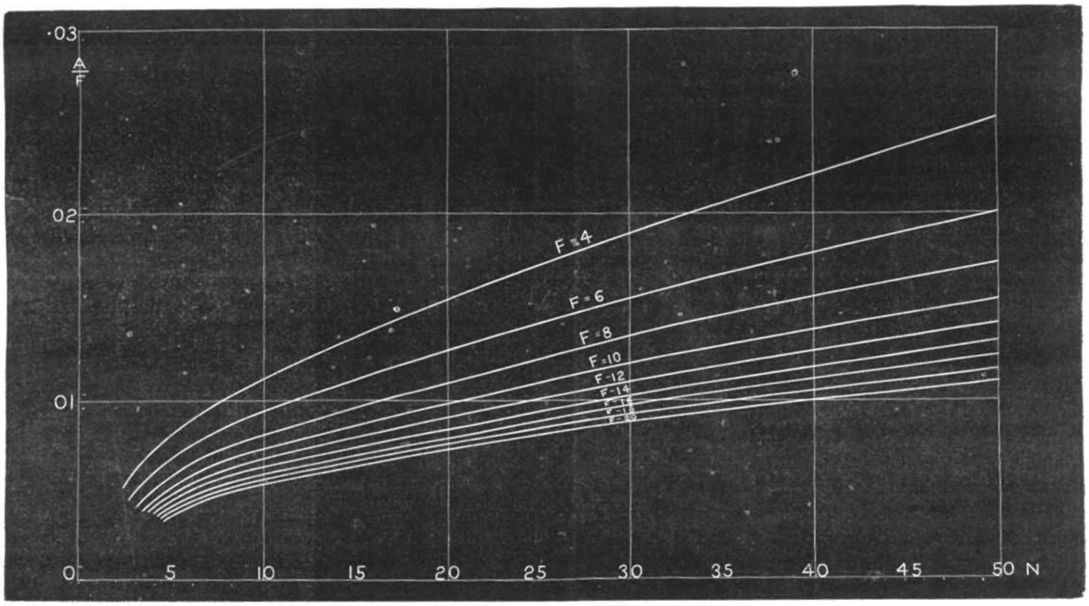

FIG. 1.

Points nearer to, or further from, the lens than that which has its image on the plate will be represented on the latter by round patches of light; these being the sections by the plate of the cones of rays which have for their summits the geometrical foci of the points, and for their slant the radius of the aperture ÷ focal length.[1] Thus, if e is the distance before or behind the plate of the focus of a point, it will be represented on the plate by a patch of light of diameter

$$e\,\frac{A}{F}.$$

This diameter can be diminished by the use of a diaphragm, *i.e.* by diminishing A, but this at the same time increases the diameter of the images of points whose foci are on the plate. And the resulting average definition will be improved by diminishing A until the patch of light, representing the point most out of focus, has the same diameter as the diffraction disk of the image point in focus.

If we suppose the photographic plate to be placed at such a distance from the lens that the focus of the nearest object is as much behind the plate as the focus of very distant objects is in front of it, we shall have, to determine the diameter of the stop giving the best average definition, the following equation:—

Putting F = principal focal length,

D = distance of nearest object,

$q = 1{\cdot}219\lambda$,

$$q\,\frac{F+e}{A} = e\,\frac{A}{F+e};$$

$$\therefore\quad q\,\frac{(F+e)^2}{e} = A^2 \;\ldots\ldots\ldots\; (1)$$

but, by the ordinary formulæ, connecting the conjugate foci of lenses, we have, if $D = F + g$,

$$2eg = F^2;$$

$$\therefore\quad e = \frac{F^2}{2g} = \frac{F^2}{2(D-F)};$$

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[1] This is an approximate statement only. The true expression involves an investigation of the intensity of the light immediately in front of and behind a caustic.

whence, substituting for e in (1) we have

$$A = \sqrt{\frac{q}{2}}\ \frac{2D - F}{\sqrt{D - F}} \quad . \; . \; . \; . \; . \; . \; . \; (2)$$

Let the nearest object be at n times the focal length of the lens. Then, putting nF for D,

$$A = \sqrt{\frac{qF}{2}}\ \frac{2n - 1}{\sqrt{n - 1}} \quad . \; . \; . \; . \; . \; . \; (3)$$

This gives the value of A as a linear quantity; it is usual, however, to reckon the diameter of stops as fractions of the focal length.

Dividing, therefore, (3) by F,

$$\frac{A}{F} = \sqrt{\frac{q}{2F}}\ \frac{2n - 1}{\sqrt{n - 1}} \quad . \; . \; . \; . \; . \; . \; (4)$$

From (4) the accompanying table has been computed, giving $\frac{A}{F}$ for various values of F and n. (Fig. 1 gives the same graphically.)

Table showing ratio of aperture to focal length which gives the best average definition when the nearest object to be photographed is at "n" times the focal length of the lens, and distant objects are also in view.

F	$\frac{A}{F}$									
	n=5	n=10	n=15	n=20	n=25	n=30	n=35	n=40	n=45	n=50
in.										
4	·00785	·0110	·0130	·0150	·0171	·0188	·0203	·0216	·0229	·0244
6	·00640	·00900	·0106	·0124	·0139	·0152	·0165	·0177	·0188	·0199
8	·00554	·00775	·0092	·0107	0121	·0132	·0142	·0154	·0162	·0172
10	·00495	·00699	·00825	·0096	·0108	·0118	·0129	·0137	·0146	·0154
12	·00452	·00635	·00755	·0087	0098	·0108	·0117	·0125	·0133	·0141
14	·00420	·00580	·00695	0081	·0091	·0100	·0108	·0116	·0123	·0130
16	·00392	·00552	·00651	·0076	·0085	·0093	·0101	·0108	·0115	·0122
18	·00370	·00520	·00615	·0071	·0080	·0088	·0095	·0102	·0109	·0115
20	·00350	·00494	·00584	0068	·0076	·0084	·0090	·0097	·0103	·0109

I have not before seen it pointed out that the ratio $\frac{A}{F}$, which gives the best average definition, alters with the value of F.

If α is the least angular distance between two points (as seen from the centre of the lens) which are shown as separate points on the photograph, α must at any rate not be less than $\frac{q}{A}$, or

$$\sqrt{\frac{2q}{F}}\ \frac{\sqrt{n - 1}}{2n - 1},$$

showing that, if the foreground is kept at a distance proportional to the focal length of the lens, the definition improves with an increase of the focal length.

On the other hand, if the nearest object is at some fixed distance, D, from the lens, we have as the limit for α,

$$\sqrt{2q}\ \frac{\sqrt{D - F}}{2D - F},$$

an expression which increases with F, so that for a given picture taken from a fixed position, definition will be gained by the use of a short focus.

The gain, however, in this respect is not great, for in practice D is always a considerable multiple of F, and writing

$$\sqrt{\frac{1}{4D + \frac{F^2}{D - F}}} \text{ for } \frac{\sqrt{D - F}}{2D - F},$$

it will be seen that when D is many times F, $\frac{F^2}{D - F}$ may be neglected in comparison with 4D.

Thus, in ordinary cases the limit for α is $\sqrt{\frac{q}{2D}}$, and is independent of the focal length of the lens employed.

If we inquire how close the nearest object may be to the lens when a view containing also distant objects has to be photographed with a definition reaching a certain standard, we have, on the above supposition,

$$D = \frac{q}{2\alpha^2};$$

and if we put $\alpha = 1'$, which is often taken as the least angle separable by the unaided eye, and λ as $\frac{1}{50000}$ inch,

$$D = 150 \text{ inches},$$

showing that if the picture is to appear as well defined as the natural objects themselves, to the eye placed at the position of the lens, no object in the view must be nearer the latter than about 13 feet.[1]

Though, as above stated, the focal length does not affect the definition, when the right-sized stop is used, it does the rapidity with which a picture may be taken, for the intensity of the light on the plate is measured by $\frac{A^2}{F^2}$ or $\frac{q}{2F}\ \frac{(2n - 1)^2}{n - 1}$.

That is, in these circumstances, the exposure is inversely as the focal length.

All that has been hitherto said refers to the definition in the central parts of the plate.

The definition for the oblique pencils is necessarily worse. For even if it were assumed that the lens was perfect for oblique pencils, the points out of focus would be no longer represented by circular areas, but by the elliptic projections of these circles on the plane of the plate.

The assumption, however, that a lens is perfect for oblique pencils is too far removed from actual fact to make it worth while to consider the results to which such a supposition would lead.

The definition for the marginal parts of the photograph depends on the various aberrations which all combinations of lenses suffer from in some degree, but which in well-made examples are completely, or almost completely, corrected for direct pencils.

These aberrations are (1) spherical, (2) chromatic, (3) astigmatism, (4) curvature of field.

The effects of the two last are much the most important, and will be considered first.

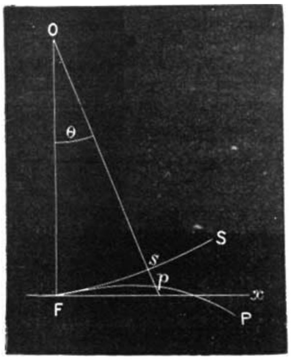

FIG. 2.

Let O (Fig. 2) be the optic centre of the lens, OF the axis of the lens, and F the principal focus.

Let Fx be the plane of the plate, FP and FS the curves on which the primary and secondary foci respectively lie.

Let Op be the axis of a pencil inclined to OF at an angle θ, and meeting FP and FS in p and s. Then sp measures the astigmatism of the lens for a pencil of obliquity θ.

Putting y_p and y_s for the ordinates of the curves FP and FS at p and s, it will be seen that a point distant θ from the axis of the lens, will be represented on the plane of the plate by an oval patch of light whose axes are $A\frac{y_p}{\cos\theta}$ and $A\frac{y_s}{\cos\theta}$ in directions parallel and perpendicular to Fx; A, as before, being the aperture of the lens.

Any formula depending on the actual data of real combinations of lenses, and giving the values of y_p and y_s in terms of radii of curvature and refractive indices, &c., of the lenses composing them, would be a very unmanageable thing for the purpose in hand; but I give the curves in question obtained experimentally for seven lenses of different types in my possession (see Figs. 3 to 9). All these lenses except Fig. 5 are by makers reputed to be the best.

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[1] I have verified this with a lens of 10-inch focus.

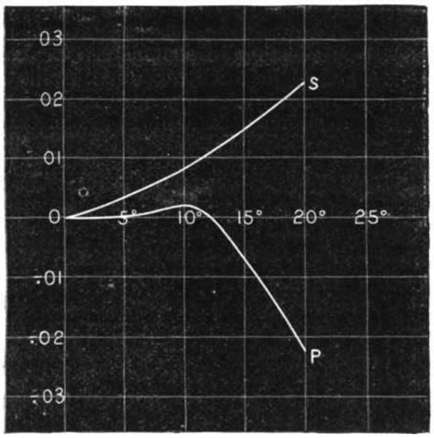

Fig. 3.—English Portrait.

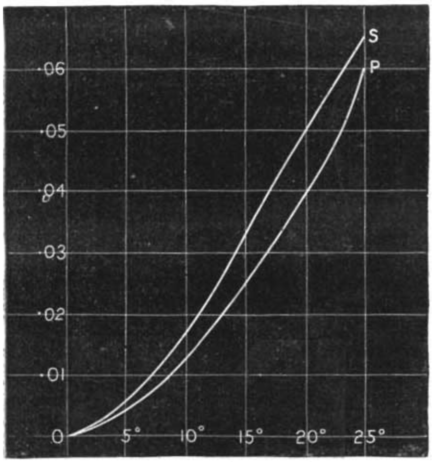

Fig. 4.—English Portrait.

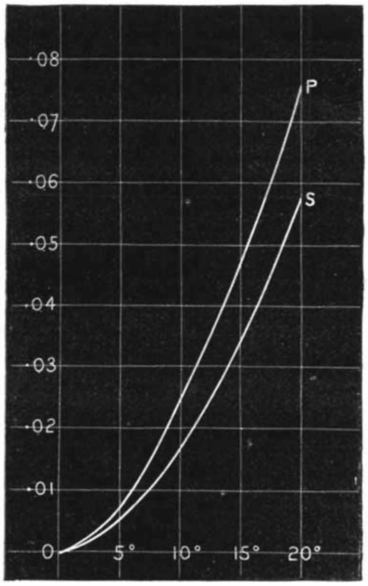

Fig. 5.—French Portrait.

[Fig. 5 is peculiar in having the usual relative positions of the primary and secondary foci reversed.]

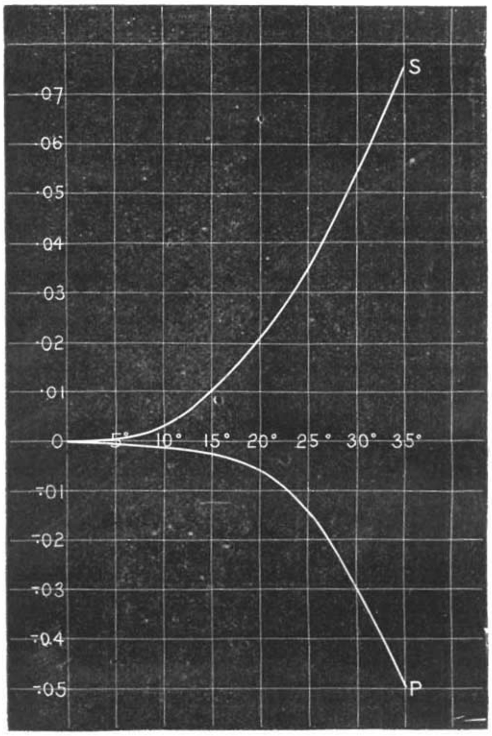

Fig. 6.—English Wide-angle Doublet.

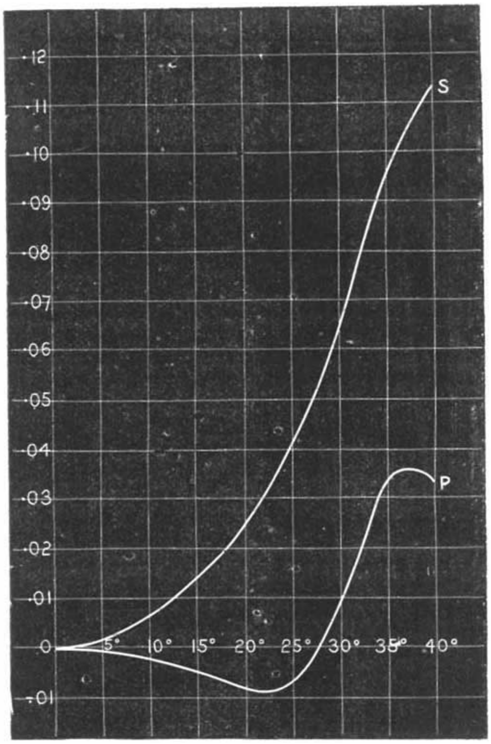

Fig. 7.—French Wide-angle Doublet.

The ordinates of these curves are the distances of the primary and secondary foci from a plane through the principal focus at right angles to the axis of the lens, and are expressed as fractions of the focal length.

The abscissæ are the inclinations (in degrees) of the pencils to the axis.

Suppose, now, that the plate is placed at a distance e behind the principal focus, e being half the distance between the foci for direct pencils of the nearest and distant objects.

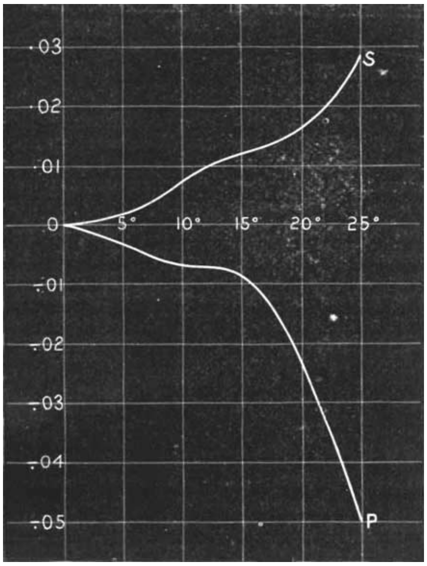

Fig. 8.—Rapid Rectilinear.

The worst defined point in the centre of the picture will then be represented as having a width $\frac{A}{F}e$ nearly, while at the obliquity θ this width becomes $\frac{A}{F}\frac{e \pm y}{\cos \theta}$ nearly, according to whether the image under consideration is that of the most distant or the nearest point.

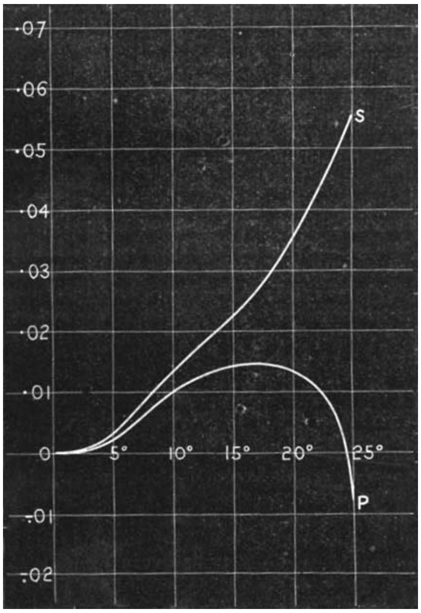

Fig. 9.—Triplet.

Hence, unless y is small compared with e, the definition for rays of obliquity θ will be sensibly worse than in the centre of the plate, and a reference to the curves for y_p and y_s shows at a glance that this must be the case even at 10° for all the lenses unless the general standard of the definition is lowered by a large value of e.

As an example of the use of the curves, let us take the rapid rectilinear No. 6, and compare the definition at 20° obliquity with that at the centre, supposing that the nearest object is at a distance of 25F.

This gives e = ·02F nearly, and at 20° $y_p = -$ ·023F, $y_s = +$ ·016 F, hence we have as follows:—

		At 20° from axis		
		due to primary focus.	due to secondary focus.	At centre.
Width of image of nearest point...	$\frac{A}{F \cos \theta} \times$ ...	·043 ...	·004 ...	·02
Distant do. ...	... ...	·003 ...	·036 ...	·02

This shows that while the nearest points at this obliquity are represented by long ovals placed as if radiating from the axis, the most distant points become similar but rather smaller ovals with their long axes at right angles to the former, and that the length of the ovals is about twice the diameter of the image formed by the direct pencils.

In the same way the definition, as far as it depends on astigmatism and curvature of field, at any obliquity may be found for any lens for which y_p and y_s are known.

Lauriston Hall, September 9. A. Mallock.

(*To be continued.*)

THE KOH-I-NUR—A CRITICISM.

THE true history of the Koh-i-Nur diamond, if it could be written, would be a singularly interesting one. But the historian would have a difficult task. The pages that I purpose writing will be devoted to the criticism, possibly the refuting, of some fallacies that hang round the subject; but they will not deal with some other historical difficulties that I have not space even to indicate, but which do not belong to those portions of the history for criticism on which the following pages are designed.

The period in the history of the Koh-i-Nur that has attracted the notice of all modern writers on the diamond, and to a degree, I think, somewhat beyond its importance, is the five or ten minutes during which the French diamond-dealer, Tavernier, held in his hand the most important of the Crown jewels of the Emperor Aurungzebe. It was a great diamond, and the record Tavernier has handed down in his "Voyages," of its weight, its form, and its history, will have to be critically dealt with.

It may be at once stated that the disputable point regarding this diamond is whether it was a certain ancient diamond of fame in India, or one much larger than this ancient stone, that had been found not very long before Tavernier was present at the Court of Aurungzebe. For the larger stone I shall retain the name of "the Great Mogul"; for the older and more famous one the title of the Koh-i-Nur. Some hold that Tavernier saw and handled the Koh-i-Nur; others that his own story is correct, and that it was the Great Mogul that he described. And I should add that some, in addition to this latter view, believe the Great Mogul ought to be called the Koh-i-Nur.

In order to clear the ground, I may say that while attaching no very great importance to the question as to which of the two first views is the correct one—and I must add also, valuing at a somewhat low estimate the historical or technical accuracy of Tavernier's statements on this and many other matters—I, some thirty-five years ago, came to the conclusion that the diamond Tavernier saw was probably the Koh-i-Nur, and that he muddled its history with the other and larger diamond that I showed to have been probably at the time in the keeping of Shah Jahan, the captive father of Aurungzebe. The merits of the question will be discussed in their proper place; but while holding myself open to conviction if any new arguments can be brought forward against my view, I may state that none yet announced have shaken that opinion.

Until the fifteenth century there appears to have been one and only one very large diamond known in India or in the world. I might have said until the sixteenth century but that there is a record of two and an unauthenticated rumour of a third during that century, the largest of which, however, was